Introduction

- Rapidly changing field:
  - vacuum tube -> transistor -> IC -> VLSI
  - memory capacity and processor speed is doubling every 1.5 years:
- Things you'll be learning:
  - Foundation of computing, design methodologies, issues in design
  - how to analyze their performance (or how not to!)
- Why learn this stuff?
  - You want to design state-of-art system
  - you want to call yourself a "computer scientist or engineer"
  - you want to build software people use (need performance)
  - you need to make a decision or offer "expert" advice

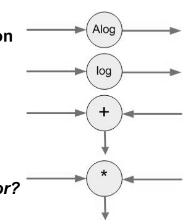
What is a computing?

- In 1960, "computer" was still understood to be a *person*
  - A person who could compute
- By contrast, a recent dictionary begins the definition as
  - A "computer" is "An electronic machine..."
- But computing has had many abstraction
- We would learn about some of them today

Consider An Example: Example 1

- Let us evaluate an expression
  - *A=B+C+D+E*F*
- *It can also be written as*
  - *A=(B+C)+D+E*F*
  - *A=(B+C+D)+E*F*
  - *A=(B+C+D)+(E*F)*
  - *A=B+(C+D)+E*F*
- *But are these correct?*
  - *A=(B+C+D+E)*F*
  - *A=B+C+(D+E*F)*
- *Depends on what are the rules for evaluating expressions*
- *What are we computing?*
- *What is the model?*

What is A Computing Abstraction?



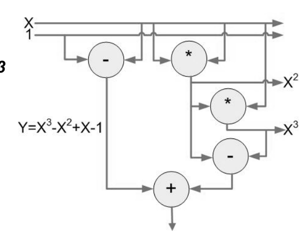
- Consider computation a simple expression
  - A=B*C
- What do we need to do to compute?
  - *Need storage for B*
  - *Need storage for C*
  - *Multiply*
  - *Need storage for A*
  - *How would you do it on your calculator?*
- What if you do not have multiplier?
- But you have black boxes that compute, add, log/alog
  - Log A = Log B + Log C
- It is a functional transformation
- How do we achieve the computation? Put the blocks together

Consider Another Example: Example 2

- Consider the computation $Y = X^3 – X^2 + X - 1$
- How many operations?
  - *How many multiply?*
  - *How many adds/subs?*
  - *How many storage?*
- Is this the best we can do?
- How do we achieve efficiency is computation?

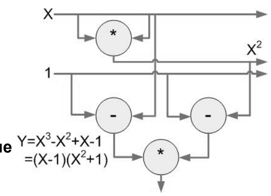
A Possible Solution: Example 2

- How many operations?
  - *How many multiply? 2*
  - *How many adds/subs? 3*
  - *How many storage?*
  - *How much time?*
- Is this the best we can do?
  - For multiplication
  - Probably we can argue
  - What about adds/subs?
- This is not very efficient



Another Possible Solution: Example 2

- **Simplify the function by factorization?** $Y=X^3-X^2+X-1$ $=(X-1)(X^2+1)$
  - ***How many multiply? 2***
  - ***How many adds/subs? 2***
  - ***How many storage?***
  - ***How much time?***



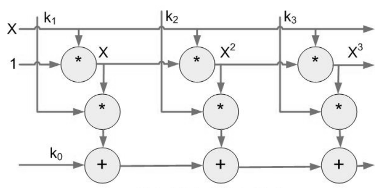
- **Is this the best we can do?**
  - **For *, probably we can argue**
  - **What about adds/subs?**
- **Another factorization does not change number of operations**

Consider One More Example: Example 3

- **Consider the computation** $Y=k_3*X^3+k_2*X^2+k_1*X+k_0$
  - **No constraints on values**
- **How many operations?**
  - ***How many multiply?***
  - ***How many adds/subs?***
  - ***How many storage?***
- **Is this the best we can do?**
- **How do we achieve efficiency is computation?**

A Possible Solution: Example 3

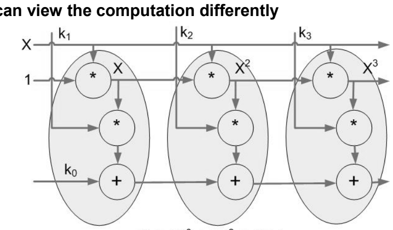
- **How many operations?**



  - ***How many multiply? 6, but one can be saved easily***
  - ***How many adds/subs? 3***
  - ***How many storage?***
  - ***How much time?***

Another Way to Solution: Example 3

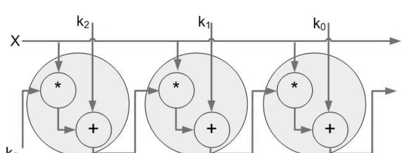
- **We can view the computation differently**



- **Why this form?**
  - **Provides a building block for computation**
- **But has each block big, 4 inputs, 2 outputs**

Yet Another Way to Solution: Example 3

- **We can look at the expression differently**



$Y=k_3*X^3+k_2*X^2+k_1*X+k_0$
$=((k_3*X+k_2)*X+k_1)*X+k_0$

- **How many operations?**
- **Why this form?**
  - **Provides a way to optimize and provides a building block**

Point of Discussion

- **A good computation structure require some thinking**
- **Optimize on hardware design cost**
- **Optimize on time for computation**
- **There may be a tradeoff that needs to be explored**
- **Identify common building blocks that can be implemented and used to realize interesting computations**
- **Always consider**
  - **How many operations?**
  - **How many time steps?**
  - **What is the tradeoff?**
  - **Solutions may not be obvious**

Computing with a designed Machine

- Consider computation in example 1 (An user may like to directly say this as is)
  - A=B*C
- A given machine has facility to load variables and perform arithmetic and complex functions (who designed it?)
- So how do we compute?
- Here is a conceptual program
  - Load B, mem1
  - Load C, mem2
  - Multiply mem1, mem2, mem3
  - Store A, mem3
- On your simple calculator
  - Key in value of B
  - Press multiply
  - Key in value of C
  - Press = and Read A out

Program Example 2

- Required computation is $Y=X^3-X^2+X-1$
- A complex program may look like
  - X = value
  - $Y = X^3 – X^2 + X – 1$
- A simple program may look like
  - Load X, mem1
  - Multiply mem1, mem1, mem2
  - Multiple mem1, mem2, mem3
  - Sub mem3, mem2, mem4
  - Add mem4, mem1, mem5
  - Load #1, mem6
  - Sub mem5, mem6, mem7
  - Store Y, mem7
- Do we need all these memory locations?

Program Example 2 Differently

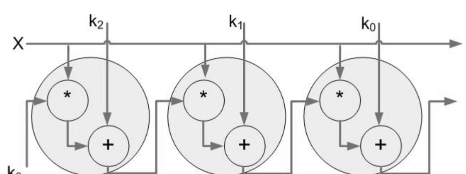
- Factorized function is $Y=X^3-X^2+X-1$
  $=(X-1)(X^2+1)$
- A simple program may look like

Now Consider Our Complex Example – 3

- Required computation is $Y=k_3*X^3+k_2*X^2+k_1*X+k_0$
- How do we approach this
- We need some structure to store variables
  - An array structure k[i], i = 0, 1, 2, 3, ...
  - A variable name X
  - Store powers of X, i.e., X^i in xpower[i], i = 0, 1, 2, 3, …
  - A result location Y
  - X is given by user
  - K[i] is filled in by user
  - Y is initially zero
  - Partial Y computation is, Y = k[0]
  - Also, xpower[0] = 1
  - At each step i = 1, 2, 3, …, we have 3 inputs and 2 outputs
    - we take xpower[i-1], partial result Y, and k[i]
    - And compute xpower[i] and a new partial result Y

Now Program Example – 3 Using Alternate

- What is the big difference?
- Block is simple, but need to start from other end



$Y=k_3*X^3-k_2*X^2+k_1*X-k_0$
$=((k_3*X+k_2)*X+k_1)*X+k_0$

Differences Between the two Programs

- First approaches computes a 3-inputs, 2-outputs function
- The second one uses a 3-input, 1-output function
  - Mathematically that is how we prefer to write functions
- First method can be used for successive addition of term
- The second method requires us to know how many terms

Computing Functions: Difference Engine

- Consider the computation $Y = X^3 - X^2 + X - 1$
- Consider the table
- What is going on each row
- Can you name each row?
- Can you tell how an entry in a row is computed?

0	1	2	3	4	5	6	7	8	9	10	11	12	13	
-1	0	5	20	51	104	185	300	455	656	909	1220	1595	2040	
1	5	15	31	53	81	115	155	201	253	311	375	445		
4	10	16	22	28	34	40	46	52	58	64	70			
6	6	6	6	6	6	6	6	6	6	6				

Difference Engine Abstraction

- Suppose you want to calculate y = Sin (x)
- Need a Sin calculator
  - Looks cheap on your calculator, it is expensive computation
  - How would you go about it?
- Consider a Taylor series expansion
  - $y = \mathrm{Sin}(x) = x - x^3/3! + x^5/5! - x^7/7! + \ldots\ldots$
- Based on computing differences, a finite n-th order polynomial can be differentiated n times, which can be represented by a difference
- What degree polynomial is sufficient?
  - Depends on accuracy needed (we will visit that many times)
- Let us consider only two terms:
  - $y = \mathrm{Sin}(x) = x - x^3/3!$

Calculating using Difference Engine

- To compute value of sin(x) at x(0), x(1), x(2), x(3), x(4), x(5), ……… such that difference in two consecutive values of x is small
  - $\Delta x = x(i+1) - x(i)$
  - $y(x(i)) = \sin(x(i)) = x(i) - x(i)^3/3!$
- For simplicity, we will drop () and denote the corresponding values of y also as y0, y1, y2, y3, …..
- We can calculate y0, y1, y2, and y3 by hand and also call them $\Delta^0 y0$, $\Delta^0 y1$, $\Delta^0 y2$, and $\Delta^0 y3$, respectively
- Why are we doing it?
- That forms the basis of difference engine abstraction

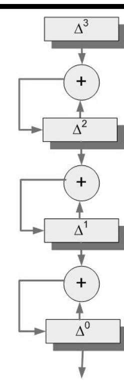
Difference Engine (cond.)

- If we differentiate the function, forth differentiation will yield a 0
- What about the third differentiation?
  - A constant (value is -1 in this case)
  - And others can be calculated as well
- First order difference can be written as
  - $\Delta^1 y0 = y1-y0$; $\Delta^1 y1 = y2-y1$; $\Delta^1 y2 = y3-y2$
- Second order difference can be written as
  - $\Delta^2 y0 = \Delta^1 y1 - \Delta^1 y0 = y2-2y1+y0$
  - $\Delta^2 y1 = \Delta^1 y2 - \Delta^1 y1 = y3-2y2+y1$
- Third order difference can be written as
  - $\Delta^3 y0 = \Delta^2 y1 - \Delta^2 y0 = y3-3y2+3y1-y0$
- And the forth order difference is $\Delta^4 y0 = 0$
- Suppose we know $\Delta^3 y0$, $\Delta^2 y0$, $\Delta^1 y0$, and $\Delta^0 y0$
- Using this we can recursively compute $\Delta^3 y1$, $\Delta^2 y1$, and $\Delta^1 y1$, and $\Delta^0 y1$
- And then all y2 and y3, and y4………

Difference Engine Example

- IN: x0 x1 x2 x3 x4 x5 x6
- OUT: y0 y1 y2 y3 y4 y5 y67
- 0^{th} Diff: $\Delta^0 y0$ $\Delta^0 y1$ $\Delta^0 y2$ $\Delta^0 y3$ $\Delta^0 y4$ $\Delta^0 y5$ $\Delta^0 y6$
- 1^{st} Diff: $\Delta^1 y0$ $\Delta^1 y1$ $\Delta^1 y2$ $\Delta^1 y3$ $\Delta^1 y4$ $\Delta^1 y5$ $\Delta^1 y6$
- 2^{nd} Diff: $\Delta^2 y0$ $\Delta^2 y1$ $\Delta^2 y2$ $\Delta^2 y3$ $\Delta^2 y4$ $\Delta^2 y5$ $\Delta^2 y6$
- 3^{rd} Diff: $\Delta^3 y0$ $\Delta^3 y1$ $\Delta^3 y2$ $\Delta^3 y3$ $\Delta^3 y4$ $\Delta^3 y5$ $\Delta^3 y6$
- In general
  - $\Delta^n y(i+1) = \Delta^n y(i)$ for nth order function and
  - $\Delta^{j+1} y(i) = \Delta^j y(i+1) - \Delta^j y(i)$ for j = 0, 1, 2, … n-1, and i = 0, 1, 2, …
  - Or $\Delta^j y(i+1) = \Delta^j y(i) + \Delta^{j+1} y(i)$ for j = 0, 1, 2, … n-1
- So if we know the values in the first column, we can compute second column and so on
- The structure need n+1 memories (to store a column) and n adders
- One can also write a C program to compute a column at a time
  - And the first column is obtained by calculating values by hand

Difference Engine Organization



Decimal System

- We are all familiar with decimal numbers
- Consider a number 2375
- What digits representing thousand, hundred, ten and one's place
- How did you get it?
- Give me an algorithm
  - Divide by 1000, result is thousand place value
  - Subtract 1000*thousand place value
  - Divide by 100, result is hundred place value
  - Subtract 100*hundred place value
  - Divide by 10, result is ten place value
  - Subtract 10*ten place value
  - Remainder is one place value
- What is good about this algorithm
- What is bad about it?

An Easier Algorithm

- Divide by 10
- Remainder is one place value
- Divide the result by 10
- Remainder is ten place value
- Divide the result by 10
- Remainder is hundred place value
- Divide the result by 10
- Remainder is thousand place value

- Any time result is zero, that means no more value

- Division is always by 10

- We always need result and remainder

Any Base b Algorithm

- Divide by b
- Remainder is one place value
- Divide the result by b
- Remainder is ten place value
- Divide the result by b
- Remainder is hundred place value
- Divide the result by b
- Remainder is thousand place value

- Any time result is zero, that means no more value

- Division is always by b

- Remainder is always between 0 and b-1

Information Representation

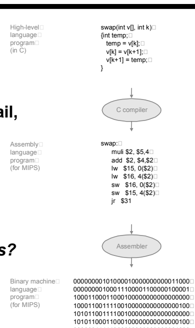
- Information theory: discusses how to deal with information
- We only deal with some aspects of it
- Virtually all computers now store information in binary form
- A binary number system has two digits, 0 and 1
- Combination of binary digits represent various kind of information
- Examples
  - 01001011
  - It can be interpreted as an integer value, a character code, a floating point number….
- Non binary numbers are also possible
- How do we represent negative numbers?
  i.e., which bit patterns will represent which numbers?

Abstraction

- Delving into the depths reveals more information

- An abstraction omits unneeded detail, helps us cope with complexity

What are some of the details that appear in these familiar abstractions?



Historical Perspective

- 1642 Pascal: Mechanical Computer
- 1671: Gottfried Leibniz ADD/SUB/MUL/DIV
- 1801: Automatic Control of Weaving Process
- 1827 The Difference Engine by Charles Babbage
- 1936: Zuse Z1: electromechanical computers
- 1941: Zuse Z2
- 1943: Zuse Z3
- 1944: Aiken: Ark 1 at Harvard
- 1942-45: ABC at Iowa State (Atanasoff-Berry Computer)
- 1946: ENIAC: Eckert and Mauchley: Vacuum Tube
- 1945 EDVAC by von-Neumann machine, father of modern computing

Why Binary?

- Easy to represent
  - Off and On
  - Open and close switch
  - Head and tail on a coin
  - Polarity of magnetization
  - 0 and nonzero voltage levels
- How to represent information in binary?
- Say we want to represent positive number 0 and 1
  - 0 is 0 and 1 is 1
- say we want to represent red and green colors
  - 0 is red and 1 is green (or vice versa)
- Say we want to represent fall and spring semesters
  - 0 is fall and 1 is spring (or vice versa)

More Complicated Examples

- Numbers 0 to 7
  - We use combination of digits
    - 1 digits gives us two combination
    - 2 will yield four
    - 3 will yield 8
  - Need three bits (binary digits)
- What if we want to represent 16 alphabets - Need four bits
- What if we want to represents numbers from 11 to 25?
- Homework Problem:
  - For each part below devise a scheme to represent, in binary, each set of symbols
    - (A) Numbers: 0, 1, 2, 3, 4, 5, 6, 7
    - (B) Alphabets: A, B, C, D, E, F
    - (C) Integers from 21 to 36

Bits and Combinations

# of Bits	# of quantities
1	2
2	4
3	8
4	16
..	..
..	..
..	..
n	2^n

- What happens in other number systems?
- In base b, n digits give b^n combinations
- Base 10: decimal
- Base 8: Octal
- Base 16: Hexadecimal

Representation of Positive Numbers

- Positional value
- Binary digits are numbered
- Right most digit is 0
- Next to that is a 1
- And so on up to n-1 in an n-bit representation
- Decimal point is implied at the right of bit 0
- Each bit is assigned a weight
- The weight of i^{th} bit is 2^i
- Using this notation
  - The value of an n bit sequence is
  - $2^{n-1} x_{n-1} + 2^{n-2} x_{n-2} + \dots + 2^1 x_1 + 2^0 x_0$
  - $= \sum_{i=0}^{i=n-1} 2^i x_i$

x x x x x x x x

7 6 5 4 3 2 1 0

Bit #	Weight
0	2^0
1	2^1
2	2^2
3	2^3

Some Examples

- Convert 0101 into decimal
  - Position: 3 2 1 0
  - Weight: 8 4 2 1
  - Digits: 0 1 0 1
  - Decimal value: 8 * 0 + 4 * 1 + 2 * 0 + 1 * 1 = 5
- Convert 10110101 into decimal
  - Position: 7 6 5 4 3 2 1 0
  - Weight: 128 64 32 16 8 4 2 1
  - Digits: 1 0 1 1 0 1 0 1
  - Decimal value: 128*1+64*0+32*1+16*1+8*0+4*1+2*0+1*1=181
- Now try 10000000
- And try 01111111

And What About the Reverse Operation?

- First see the largest weight of a binary positional digit contained in the number
- Put that binary digit = 1 and subtract weight
- Then try subtracting the next bit's weight
- If successful
  - next bit is 1, else next bit is 0 (and restore the value)
- Repeat the last two steps until done
- Convert decimal number 181 into binary
- Largest weight is 128, subtract 128 and set bit 7 = 1
- Try subtracting 64 out of remainder 53 (181-128)
- No successful, so the next digit is 0
- Try weight 32, 16, 8, 4, 2, and 1 successively
- Number is 1 0 1 1 0 1 0 1

A Simpler Method

- Convert decimal number 181 into binary
- Start dividing by 2
- Successive remainders are digits from right
- 181/2 = 90 remainder 1
- 90/2 = 45 remainder 0
- 45/2 = 22 remainder 1
- 22/2 = 11 remainder 0
- 11/2 = 5 remainder 1
- 5/2 = 2 remainder 1
- 2/2 = 1 remainder 0
- 1/2 = 0 remainder 1
- Number is 1 0 1 1 0 1 0 1

And Now Try Some Problems

- Suppose you want to represent positive integers in binary.
- Indicate how many bits are required to represent each of the following sets of integers:
  - (1) The integers from 0 to 127 inclusive
  - (2) The integers from 0 to 2,048 inclusive
  - (3) The integers from 0 to 32,500 inclusive
  - (4) The integers from 0 to 1,500,345 inclusive
- Indicate how large a value can be represented by each of the binary quantities: A (1) 4-bit, (2) 12-bit, and (3) 24-bit quantity.
- Convert each of the following binary digits into decimal. Assume these quantities represent unsigned integers.
  - (1) 1010; (2) 10010; (3) 0111110; (4) 10000000; (5) 0111111
- Convert each of the following decimal numbers into binary.
  - (1) 6; (2) 13; (3) 111; (4) 147; (5) 511

Base 'b' number

- In general a number system can have any base b
- the digit used are 0, 1, … , b-1
- The weight of i^{th} place is b^i
- The conversion formula from base b into decimal number is

$$\sum_{i=0}^{i=n-1} b^i x_i \quad \text{for i = 0 to n – 1} \quad \text{for an n digit quantity}$$

- Commonly used base are 2, 3, 8, 10, 16, ...

Bases 2, 8, and 16 are related

Binary	Decimal	Octal	Hexadecimal
0000	00	00	0
0001	01	01	1
0010	02	02	2
0011	03	03	3
0100	04	04	4
0101	05	05	5
0110	06	06	6
0111	07	07	7
1000	08	10	8
1001	09	11	9
1010	10	12	A
1011	11	13	B
1100	12	14	C
1101	13	15	D
1110	14	16	E
1111	15	17	F

Conversion

- From binary to octal
  - make groups of 3 bits from right to left

  $01\ 110\ 110_2 \Rightarrow 166_8$
- From octal to binary
  - make each digit as 3 bits sequence

  $276_8 \Rightarrow 010\ 111\ 110_2$
- From binary to hexadecimal
  - make groups of 4 bits from right to left

  $0111\ 0110_2 \Rightarrow 76_{16}$
- From hexadecimal to binary
  - make each digit as 4 bits sequence

  $37_{16} \Rightarrow 0011\ 0111_2$

Signed numbers

- Positive numbers are well understood
- An n-bit number represents numbers from 0 to 2^n-1
- n+m bits can be used to represent n-bit integer and m-bit fraction of a number
- However negative numbers cause another problem
- In all solutions, one bit is needed to represent the sign, + or -
- MSB can be used for that purpose, i.e., represent sign
- Remaining bits can be interpreted differently
  - They can represent magnitude as a positive number
  - They can be complemented (represent 0 by 1 and 1 by 0)
  - Or manipulate in some other way